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Influence of Turbulent Boundary Layer on Shock Tube Test Time

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Nomenclature

τ = real test time at fixed observation point
τ_i = ideal test time = $(x_s/U_s)(W-1)^{-1}$
x_s = distance from diaphragm to observation point
d = hydraulic diameter of shock tube
U_s = shock front velocity
M_s = shock Mach number
ρ = density
W = ρ_2/ρ_1
p = pressure
T = temperature
a = speed of sound
ν = kinematic viscosity
Re = Reynolds number = (ad/ν)

Subscripts

1 = initial condition, driven section
2 = flow region behind shock front
3 = flow region behind contact front
4 = initial condition, driver section
s = condition on shock front
w = condition on the wall

IN a recent publication, Roshko and Smith[1] used a formula given by Mirels[2] to compute test times in a shock tube and to compare them with experimental results. The experiments were conducted on the GALCIT 17-in. shock tube using helium as driver gas, and air and argon, respectively, as driven gas.

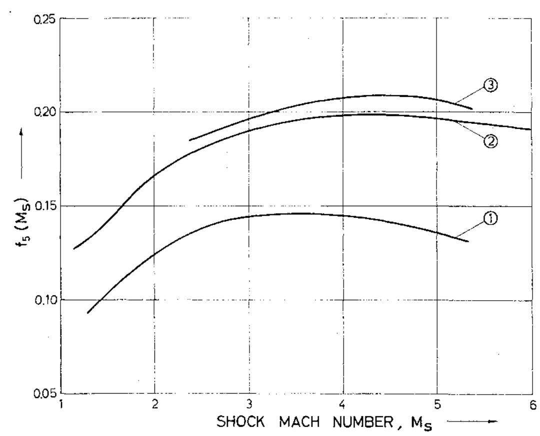

Fig. 1 Function $f_5(M_s)$ for special conditions: ① driver gas air, driven gas air; ② driver gas helium, driven gas argon; ③ driver gas helium, driven gas air; and $T_1 = T_4 = 293°K, T_{2,w} = T_{3,w} = 293°K$ in each case.

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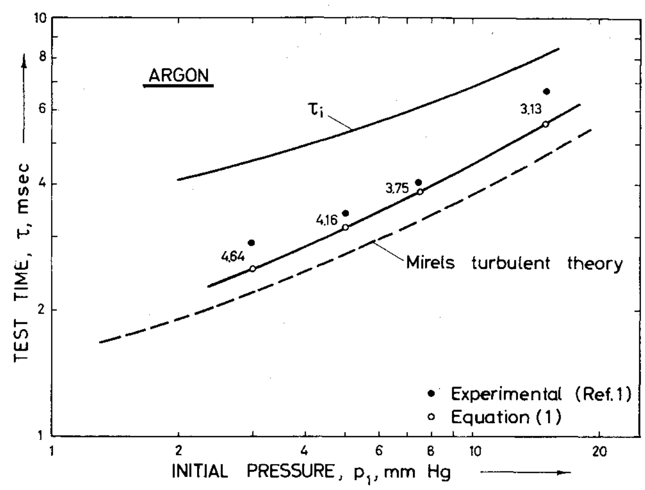

Fig. 2 Test time τ vs initial pressure p_1; driver gas helium, driven gas argon; shock Mach number indicated for each experimental point.

In the range of turbulent theory, the agreement between calculated and measured test times is good for air as driven gas, but for argon the computed times are less than the measured ones by about 40%. This discrepancy increases even more if one takes the diaphragm opening time into account.

The purpose of this note is to present an alternative to the comparison carried out by Roshko and Smith. Their experimental results are compared with test times computed from a relation given by Brocher [see Eq. (17) of Ref. 3]

$$\tau/\tau_i = 1 - f_5(M_s) \times Re_1^{-0.2} \times (x_s/d)^{0.8} \qquad (1)$$

In this formula, all the first-order perturbation terms due to the viscous effects are retained. In Fig. 1, the function $f_5(M_s)$ is represented for several conditions of interest. In Figs. 2 and 3, test times computed from Eq. (1) are compared with theoretical predictions based on the relation given by Mirels[2] and the experimental results from Ref. 1.

Test times calculated from Eq. (1) fit well the experimental data for argon, whereas for air only those for $M_s \leq 3.3$ are more accurate than values obtained from Mirels' formula. Equation (1) is particularly satisfying for small shock Mach numbers M_s. This may be so because Eq. (1) is a linear perturbation formula and may no longer be valid for large M_s, where $\tau/\tau_i \approx 0.5$.

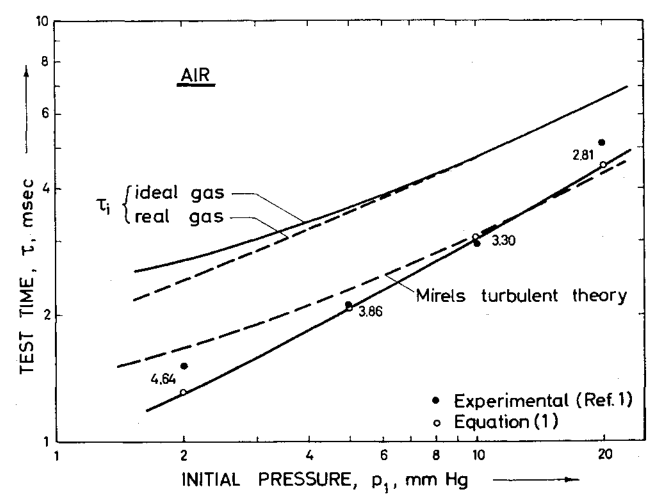

Fig. 3 Test time τ vs initial pressure p_1, driver gas helium, driven gas air; shock Mach number indicated for each experimental point.

The failure of Mirels' theory to give better results is ascribed to restrictions inherent in his local similarity approach. Only perturbations in u_2 are taken into account, whereas variations in U_s, p_2, and ρ_2 are neglected.

Remaining discrepancies may be explained by other parameters not included in either of the two theories, for example, the diaphragm opening mechanism.

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Inadequacy of Nodal Connections in a Stiffness Solution for Plate Bending

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In choosing element displacement functions for a stiffness method of analysis the following criteria must be met:

1) It must be possible to represent the rigid body motions of an element. Otherwise the equilibrium conditions of the element as a whole are falsified.[1]

2) It must be possible to represent states of constant stress. Otherwise, as the mesh of elements is finely subdivided, there is no guarantee that the stresses will converge toward continuous functions; in general, they will not converge at all.[2]

3) Where neighboring elements abut between nodes, there must be no discontinuity of slopes and deflections between the two elements.[3] Otherwise the idealization includes hinges or sawcuts between elements as well as the constraints imposed by the displacement functions. Therefore the bound theorems no longer hold[1, 2, 4] and the solution cannot be described as "pure stiffness."

This note shows that 2 and 3 are incompatible for plate elements in bending, which implies that elements should not be tied at nodes in plate and shell problems but should be matched along boundaries.[5]

Triangle ABC in Fig. 1 is given unit rate of twist about AB, so that at every point $w = xy$. Thus the quantity known as "Torsion" $= \partial^2 w/\partial x \partial y = w_{xy}$, is unity at every point in ABC. It is an easy matter to calculate the nodal rotations θ_1, θ_2, and θ_3 in the directions of the sides, and the nodal deflection δ. At least one of these, applied alone, must give nonzero w_{xy} at A. Say it is δ, θ_2, or θ_3 (or a combination of them) which gives $w_{xy} = K \neq 0$. Consider any undistorted triangle ABC', which is tied to ABC distorted at C. Near A, in ABC

$$\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial x \partial y} \times dx = K\,(dx) \tag{1}$$

whereas in ABC' it is zero. The nonconformity of slope, required to be zero along AB, apparently has a nonzero derivative along AB; this is a contradiction.

Suppose, however, it is θ_1 that gives $w_{xy} = K \neq 0$ at A. Let $AC'B$ be a reflection of ACB about AB, and let both be distorted by θ_1 only. By symmetry, if the deflections on AB are to conform they must be zero, i.e., AB cannot move. Also AC cannot move, otherwise an undistorted triangle ADC could not conform. It follows that, near A, $w = K_1 \times$ (distance from AB) $\times$ (distance from AC) where K_1 is chosen to give the xy term a coefficient K. It follows as before that slope conformity with triangle ADC is impossible.

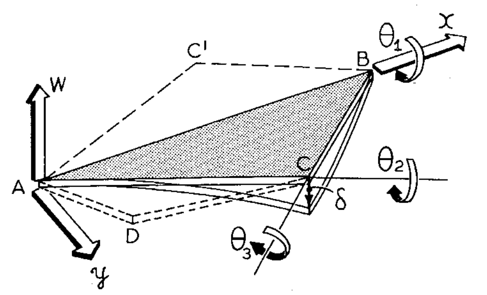

Fig. 1. Illustration of torsional deformation of element ABC with attached triangle.

These arguments extend without difficulty to the polygonal element. If the displacement functions have singularities at the nodes, such that w_{xy} has no unique limiting value, this proof fails.

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Structural Eigenvalue Problems: Elimination of Unwanted Variables

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The tendency in structural analysis is to use hundreds or thousands of variables, whereas the processes involved in finding eigenvalues of full matrices favor tens of variables, or at most slightly over 100. In frequency calculations the classic (but inefficient) technique is to use discrete masses associated with certain selected deflections. A better technique in simple cases is to use fewer elements and to write the kinetic energy and the strain energy in terms of the same assumed deflected shape, with distributed mass. However, engineers do not usually divide a structure into many elements if it can be avoided.

The proposed method uses distributed mass in the KE but retains only a small proportion of the nodal deflections, hereafter termed "masters." The remaining "slave" deflections take values giving least strain energy, regardless of what this does to the KE. Thus a slave node is assumed free from inertial forces. The argument is most clearly visualized in the case of a cantilever. If one takes a result from a discrete mass calculation, draws a smooth curve through the deflected points, and recalculates the KE from the curve, the natural frequency can be corrected to give tolerably good answers. The practical engineer may use interpolation formulas or french curves, or he may prefer to use a flexible beam to draw his smooth curve. But the best flexible beam to use would be the cantilever itself, especially if it had dis-

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